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ABSTRACT

A simple method for estimating redesignation capability in LEM descent was developed. A linear final approach trajectory is perturbed by changing the thrust vector direction and magnitude in an initial phase. Subsequently, the original end conditions of velocity and altitude are achieved by reverting to a linear path. The horizontal displacement of the end point, up or downrange, or crossrange, from the original end point is computed. The resulting flight time and the ΔV cost are also computed. The effects of such constraints as a thrust magnitude limit and impingement of line of sight to the end point on the window boundary are shown.

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SUBJECT: A Simple Method for Estimating Redesignation Capability in LEM Descent - Case 320

DATE: April 26, 1966

FROM: F. Heap

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TECHNICAL MEMORANDUM

1. Introduction

The LEM descent trajectory generated by the MIT guidance equations is a gradually curving path, concave downwards, with the end point about 75 feet directly above the intended touchdown point, horizontal velocity zero, rate of descent about 5 ft/second. Redesignation of the landing point is effected by changing the coordinates of the end point in the guidance scheme and recalculating the acceleration profile (which is a quadratic function of time). The time to go is also recomputed to satisfy a condition that the time derivative of acceleration at the end point be a constant.

In order to gain some insight into the ΔV cost of redesignations in Phase II of the descent trajectory and the effects of redesignation on look angles, a simple scheme of perturbing a constant flight path angle, constant acceleration, and constant pitch angle trajectory was developed. A flat non-rotating, constant gravity moon model was assumed. Two distinct classes of designation were considered, up and downrange and crossrange.

2. Computational Scheme

2.1 Up-Downrange Corrections

The correction maneuver is shown on the schematic of Figure 1. The unchanged trajectory is a straight path from the start to the original end point, with pitch angle, θ, and the thrust/mass ratio, (T/m), constant. The corrected trajectory is in two segments. In Phase A, for a given time interval, Δt_A, the pitch angle, θ_A, and the thrust/mass ratio $(T/m)_A$, have constant values changed from the original values. This effects a change in the flight path angle to γ_B. For

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Phase B, the thrust/mass ratio, $(T/m)_B$, the pitch angle, θ_B, and the time interval, Δt_B, are recomputed to achieve the same final conditions of altitude and velocity as in the original end point.

The equations of motion are:

$$mV\dot{\gamma} = T\cos(\theta-\gamma) - mg\cos\gamma \tag{1}$$

$$m\dot{V} = -mg\sin\gamma - T\sin(\theta-\gamma) \tag{2}$$

where

m	= vehicle mass	(slugs)
V	= instantaneous velocity	(ft/sec)
γ	= flight path angle	(radians)
T	= rocket thrust	(lb)
θ	= pitch angle	(radians)
g	= lunar gravity	(ft/sec^2)

For Phase A, the thrust/mass ratio $(T/m)_A$ and the pitch angle, θ_A, are chosen, as well as the γ_B which is to be achieved. Assume that $\gamma_{mean} = (\gamma_u+\gamma_B)/2$, $\dot{\gamma} = \frac{\gamma_B-\gamma_u}{\Delta t_A}$ and estimate a V_{mean}. (Subscript u refers to the uncorrected trajectory). Then from equation (1),

$$\theta_A = \gamma_{mean} + \arccos\left\{\frac{V_{mean}\,\dot{\gamma} + g\cos\gamma_{mean}}{(T/m)_A}\right\} \tag{3}$$

From equation (2),

$$\dot{V}_A = -g\sin\gamma_{mean} - \left(\frac{T}{m}\right)_A \sin\left(\theta_A-\gamma_{mean}\right) \tag{4}$$

and

$$V_{mean} = V_o + \dot{V}_A\,\Delta t_A/2 \tag{5}$$

(Subscript o indicates initial conditions). Equations (3), (4), and (5) are iterated three or four times until there is negligible change in $\dot{V}_A$ between iterations.

The conditions at the end of Phase A, (subscript 1) are obtained as follows:

$$V_1 = V_o + \dot{V}_A \, \Delta t_A \tag{6}$$

$$h_1 = h_o + V_{mean} \sin\gamma_{mean} \, \Delta t_A \tag{7}$$

$$x_a = V_{mean} \cos\gamma_{mean} \, \Delta t_a \tag{8}$$

where h is altitude and x is horizontal distance. The time interval for Phase B (Δt_B) is given by the altitude to be lost divided by the mean rate of descent (which is linear, as the acceleration along the flight path is again constant). That is,

$$\Delta t_B = -2(h_1-h_f)/(V_1+V_f) \sin\gamma_B \tag{9}$$

(Subscript f indicates final conditions.) The thrust/mass ratio, $(T/m)_B$, for Phase B is given by solving equations (1) and (2) simultaneously, (with $\dot{\gamma} = 0$).

$$(T/m)_B = \left[\left\{ (V_1-V_f)/\Delta t_B - g \sin\gamma_B \right\}^2 + \left\{ g \cos\gamma_B \right\}^2 \right]^{\frac{1}{2}} \tag{10}$$

Then the pitch angle, θ_B, for Phase B is given by equation (1), with $\dot{\gamma} = 0$:

$$\theta_B = \gamma_B + \arccos \left| g \cos\gamma_B/(T/m)_B \right| \tag{11}$$

The horizontal distance in Phase B is given by

$$\Delta x_B = -(h_1-h_f)/\tan\gamma_B \tag{12}$$

The distance corrected is then

$$\Delta x_{corrected} = \Delta x_A + \Delta x_B + \frac{(h_o-h_f)}{\tan\gamma_A} \tag{13}$$

The ΔV cost is given by

$$\Delta V_{cost} = \left(\frac{T}{m}\right)_A \Delta t_A + \left(\frac{T}{m}\right)_B \Delta t_B - \left(\frac{T}{m}\right)_u \Delta t_u \qquad (14)$$

where subscript u again refers to the uncorrected trajectory.

Characteristics for the uncorrected trajectory can be obtained by assuming final conditions of velocity and altitude (V_f and h_f), flight path angle γ_u and pitch angle θ_u, and solving equations (1) and (2) with $\dot{\gamma} = 0$. Starting conditions can then be obtained by assuming an altitude or distance, or a time from the final conditions.

2.2 Crossrange Corrections

The correction maneuver is shown schematically in Figure 2. The uncorrected trajectory is a straight path as in the previous section. The corrected trajectory is again in two segments. In Phase A, the vehicle is rotated about the Z axis (pilot roll) through a "bank angle". This causes a rate of change of azimuth angle $\dot{\psi}$, and the vehicle develops an azimuth angle ψ_B in some time Δt_A. In order to maintain the flight path elevation angle, γ, constant, the thrust/mass ratio $(T/m)_A$ has to be increased. This is a simplifying assumption made to keep the crossrange analysis separate from the up-downrange correction. (In practice, both corrections could be combined, but our aim here is to keep the calculation simple.) For Phase B, $(T/m)_B$, θ_B and Δt_B are unchanged from the basic trajectory.

The equations of motion are:

$$mV \cos\gamma \, \dot{\psi} = T \sin\phi \qquad (15)$$

$$mV\dot{\gamma} = T \cos\phi \cos(\theta-\gamma) - mg \cos\gamma \qquad (16)$$

$$m\dot{V} = - mg \sin\gamma - T \cos\phi \sin(\theta-\gamma) \qquad (17)$$

where

ψ = azimuth angle (radians)

ϕ = bank angle (radians)

and the other symbols have the same meanings as previously defined.

For Phase A, choose a bank angle, ϕ_A, and an azimuth angle to be turned through, (ψ_B). $\dot{\gamma}$ is to be zero, and θ and γ are to be unchanged. From equation (16), with $\dot{\gamma} = 0$,

$$(T/m)_A = g \cos\gamma / \cos\phi_A \cos(\theta-\gamma) \tag{18}$$

$$= (T/m)_u / \cos\phi_A \tag{19}$$

where subscript u again refers to the uncorrected trajectory. Note that (19) and (17) show that $\dot{V}$ will remain constant, independent of ϕ. It can be evaluated at $\phi = 0$.

From (15),

$$\dot{\psi} = \left(\frac{T}{m}\right)_A \frac{\sin\phi_A}{V \cos\gamma} \tag{20}$$

Substituting for $(T/m)_A$ from (18)

$$\dot{\psi} = \frac{g \cos\gamma \sin\phi_A}{\cos\phi_A \cos(\theta-\gamma) V \cos\gamma} = \frac{g \tan\phi_A}{V \cos(\theta-\gamma)} \tag{21}$$

We can now find an expression for V in terms of ψ.

$$\frac{dV}{d\psi} = \frac{\dot{V}}{\dot{\psi}} = \frac{\dot{V} V \cos(\theta-\gamma)}{g \tan\phi_A} \tag{22}$$

Writing a_p for the constant value of $\dot{V}$ and rearranging,

$$\frac{dV}{V} = \frac{a_p \cos(\theta-\gamma)\, d\psi}{g \tan\phi_A} \tag{23}$$

or

$$\log_e \frac{V}{V_o} = \frac{a_p \cos(\theta-\gamma)\psi}{g \tan\phi_a} \qquad (\psi_o = 0) \tag{24}$$

or

$$V = V_o \exp\left\{\frac{a_p \cdot \cos(\theta-\gamma)\psi}{g \tan\phi_A}\right\} \tag{25}$$

The crossrange distance (and in a similar way, the downrange distance) gained during the turn can be derived in terms of the azimuth angle ψ as follows:

$$\dot{y} = V \cos\gamma \sin\psi \tag{26}$$

$$\frac{dy}{d\psi} = \frac{\dot{y}}{\dot{\psi}} = \frac{V^2 \cos\gamma \sin\psi \cos(\theta-\gamma)}{g \tan\phi_A} \tag{27}$$

Substituting for V from (25),

$$\frac{dy}{d\psi} = V_o^2 \exp\left\{\frac{2 a_p \cos(\theta-\gamma)\psi}{g \tan\phi_A}\right\} \cdot \frac{\cos\gamma \cos(\theta-\gamma) \sin\psi}{g \tan\phi_A} \tag{28}$$

$$\Delta y = \frac{V_o^2 \cos\gamma \cos(\theta-\gamma)}{g \tan\phi_A} \int_o^{\psi} e^{b\psi} \sin\psi \, d\psi \tag{29}$$

where $b = 2 a_p \cos(\theta-\gamma)/g \tan\phi_A$.

$$\Delta y_B = \frac{V_o^2 \cos\gamma \cos(\theta-\gamma)}{g(b^2+1) \tan\phi_A} \left\{e^{b\psi_B}(b \sin\psi_B - \cos\psi_B) + 1\right\} \tag{30}$$

Similarly,

$$\Delta x_B = \frac{V_o^2 \cos\gamma \cos(\theta-\gamma)}{g(b^2+1) \tan\phi_A} \left\{e^{b\psi_B}(b \cos\psi_B + \sin\psi_B) - b\right\} \tag{31}$$

The time spent turning and the altitude at the end of the turn are given by:

$$\Delta t_A = (V_1-V_o)/a_p \tag{32}$$

where V_1 is the velocity at the end of the turn given by using ψ_B in equation (25), and

$$h_1 = h_o + (V_o+V_1)\Delta t_A \sin\gamma/2 \tag{33}$$

It now remains to compute the crossrange and downrange distances covered in Phase B.

They are given quite simply by:

$$\Delta y_B = (h_f - h_1) \sin\psi_B / \tan\gamma \tag{34}$$

$$\Delta x_B = (h_f - h_1) \cos\psi_B / \tan\gamma \tag{35}$$

The ΔV cost is that incurred by the increased (T/m) ratio in Phase A, as the total time for the combined Phases A and B is the same as for the uncorrected trajectory, because of $\dot{V}$ and γ being constant.

$$\Delta V_{cost} = \left\{ \left(\frac{T}{m}\right)_a - \left(\frac{T}{m}\right)_u \right\} \Delta t_A \tag{36}$$

$$= \left(\frac{T}{m}\right)_u (\sec\phi_A - 1)\, \Delta t_A \tag{37}$$

3. Discussion of Results

3.1 Up-Downrange Corrections

Calculations were made for designation capability up and downrange from a basic trajectory having $\theta = 45°$, $\gamma = -12°$, final altitude 1000 ft, final velocity 30 ft/sec. A value of 20 seconds was assumed for Δt_A. Three starting conditions were assumed, altitudes of 10,000, 6000 and 4000 feet, corresponding to distances of approximately 7 n.m., 4 n.m. and 2-1/4 n.m., respectively. The results are shown in figures 3 to 5.

ΔV cost is shown as a function of the distance corrected, for various values of θ_a. Lines of constant γ_b are shown for reference. The hatched lines are limit lines of 60% thrust in Phases A and B (100% thrust in Phase A is also shown), and lines of $(\theta_A - \gamma_m) = 65$ deg and $(\theta_B - \gamma_B) = 65$ deg. With current engine constraints, the thrust level may not be increased above 60% once reduced from full throttle. The 65 degree limit on $(\theta - \gamma)$ is when the line of sight to the end point intersects the bottom of the LEM window. This is

a temporary condition in Phase A so the limit line is shown dashed, but the Phase B limit line should not be crossed. Another boundary to the operating region is the ΔV available. It is shown tentatively as 120 ft/sec on the figures.

The extremes of the up-downrange designation capability are tabulated below:

Start Altitude (ft)	Start Distance	Distance (ft) Uprange	Distance (ft) Downrange
10000	7 n.m.	7000	13000
6000	4 n.m.	5000	10000
4000	2-1/4 n.m.	2500	8000

3.2 Crossrange Corrections

Calculations were made for crossrange designation capabilities from the same basic trajectory as before, that is, $\theta = 45°$, $\gamma = -12°$, final altitude 1000 feet, final velocity 30 ft/sec. One starting condition, altitude 4000 feet, distance 2-1/4 n.m., was considered. The results are shown on figure 6.

ΔV cost is shown as a function of crossrange distance correction, for various values of bank angle ϕ_A and azimuth angle ψ_B. The hatched line is the 60% thrust limit line (Phase A). Also shown is the time taken in the turn and the altitude at the end of the turn.

As all of the ΔV cost is incurred in the turn, the plot can also be used to find the cost of a turn through a given azimuth angle in a given time, for instance, without regard to the crossrange distance covered in the whole descent.

4. Summary

While the calculation scheme developed is not completely representative of the correction maneuvers that the LEM guidance scheme will generate, there is sufficient similarity that the results obtained can be useful in estimating the redesignation capability of the LEM in the final approach phase. The computations are simple and results can quickly be obtained using a slide rule.

The results show that uprange correction capability is limited by the amount of thrust available and by loss of line of sight to the landing point. Downrange designations are only limited by the amount of fuel available, which was to be expected. Crossrange capability is similarly unlimited except by fuel considerations.

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F. Heap

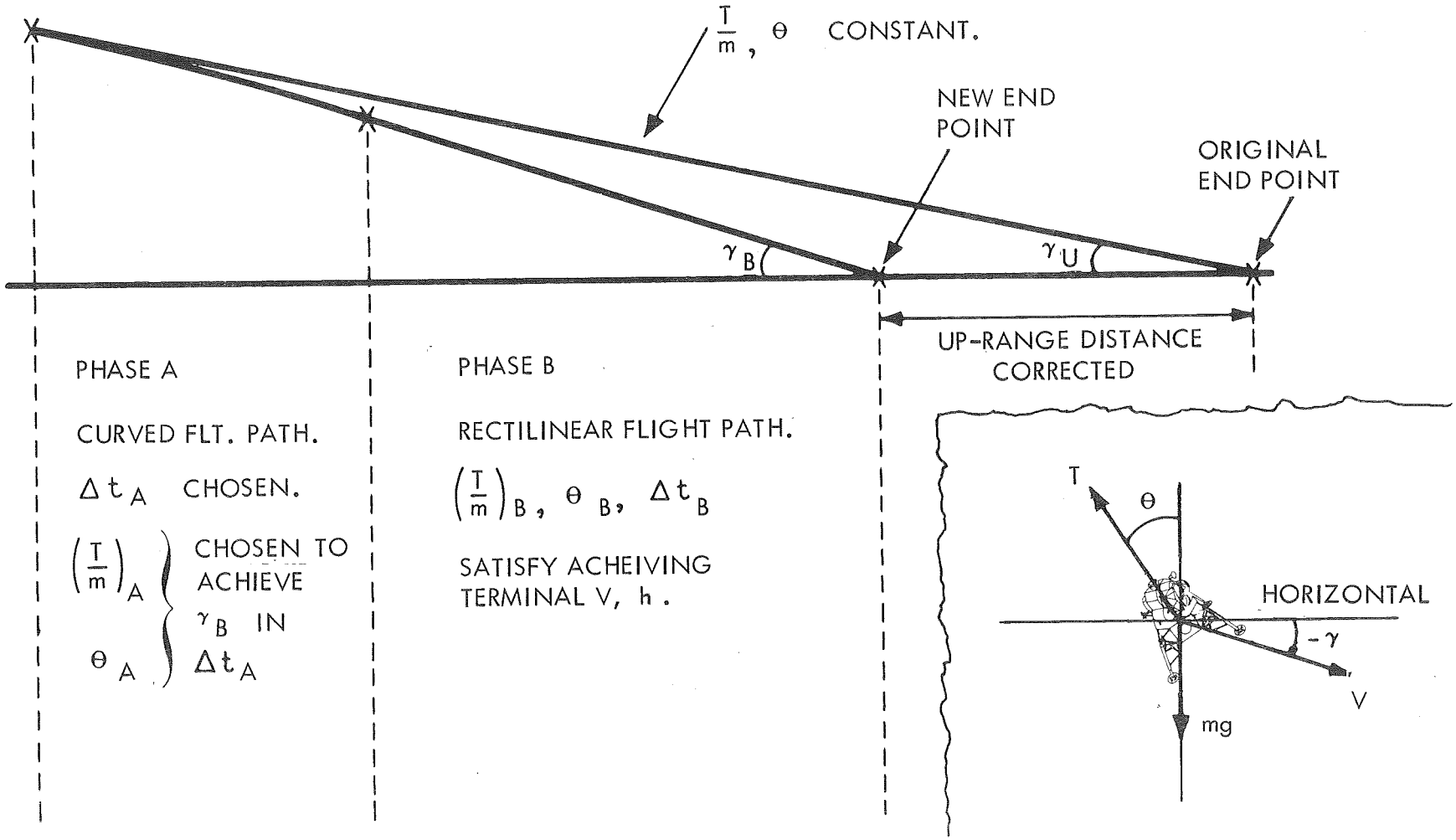

FIGURE 1 UP/DOWN RANGE LANDING POINT CORRECTION.
(RECTILINEAR FLIGHT PATH)

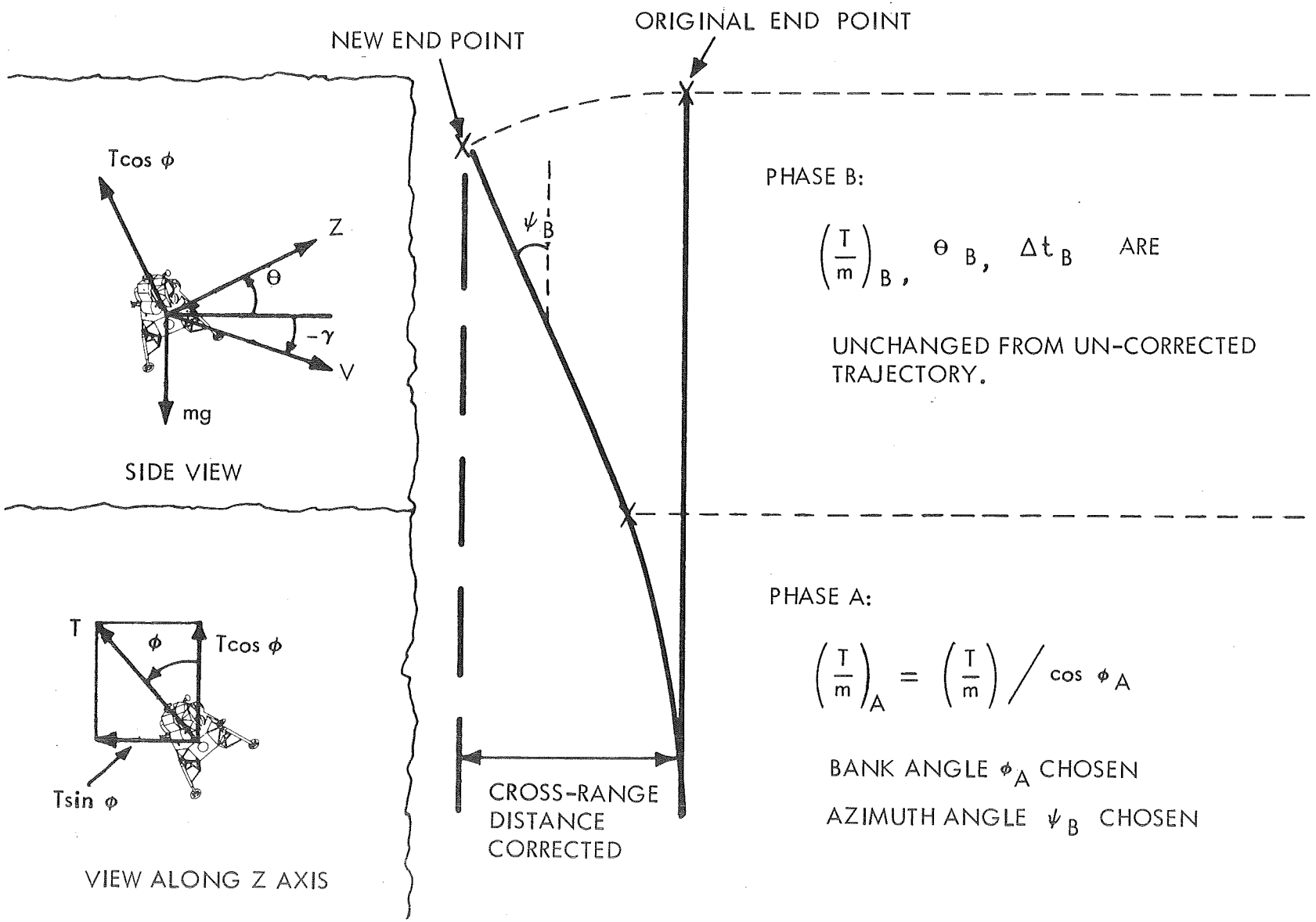

FIGURE 2 CROSSRANGE LANDING POINT CORRECTION

INITIAL CONDITIONS: DISTANCE 7 NAUT. MILES
ALTITUDE 10000 FT.
FLIGHT PATH ANGLE -12 DEG
PITCH ANGLE 45 DEG

TIME IN PHASE A 20 SECS.

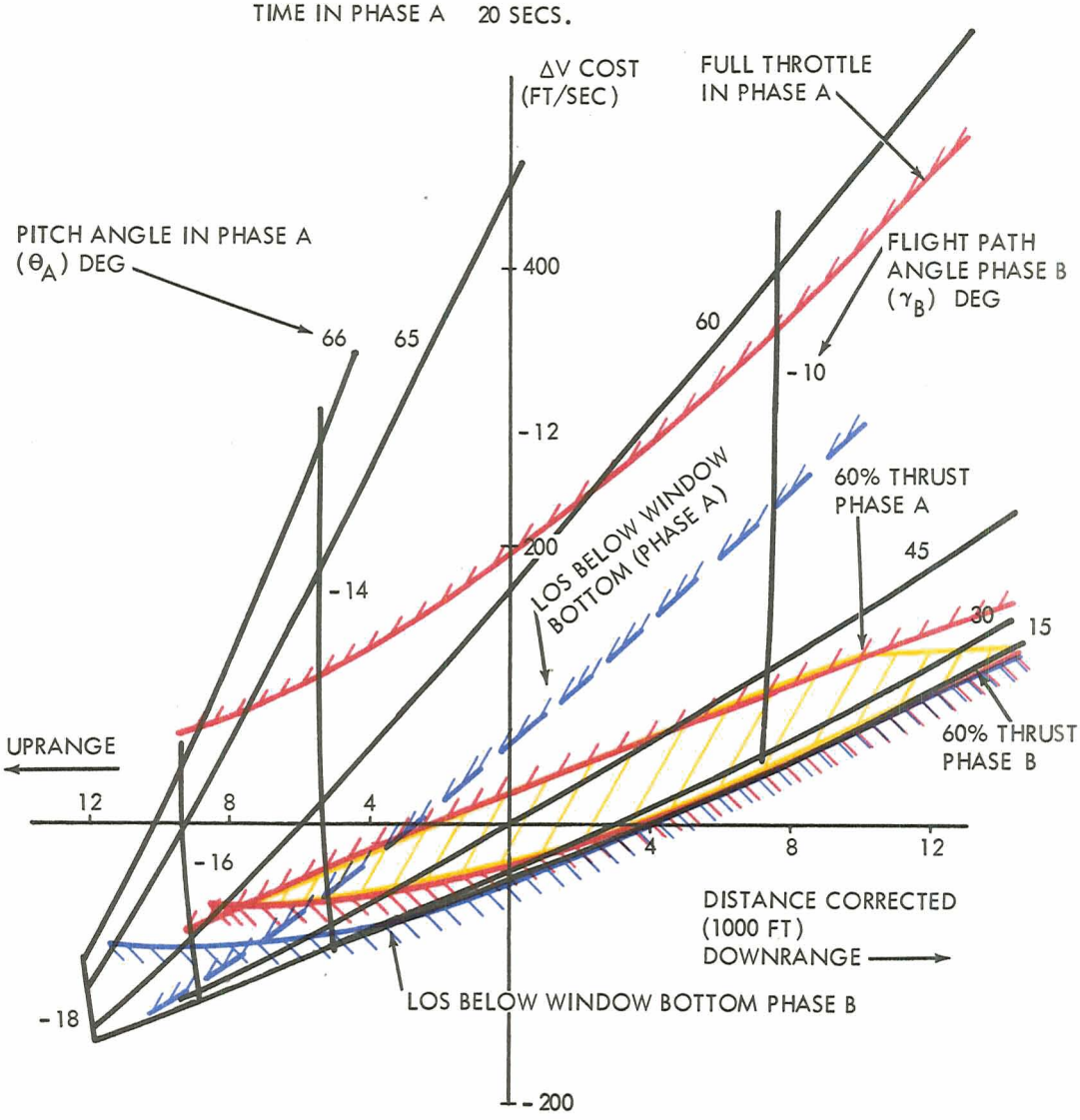

FIGURE 3 UP- AND DOWN-RANGE CORRECTIONS IN LEM DESCENT

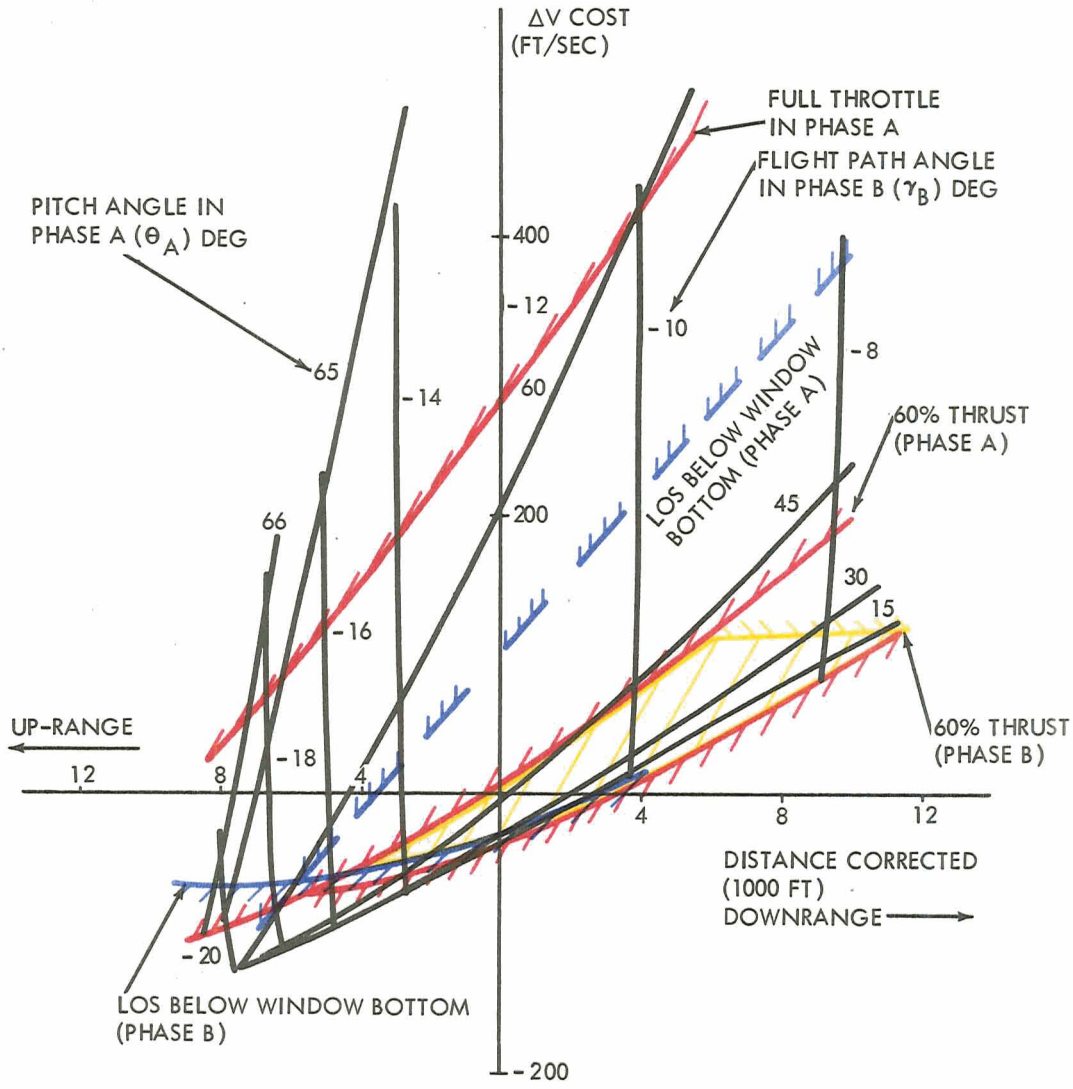

FIGURE 4 UP- AND DOWN-RANGE CORRECTIONS IN LEM DESCENT

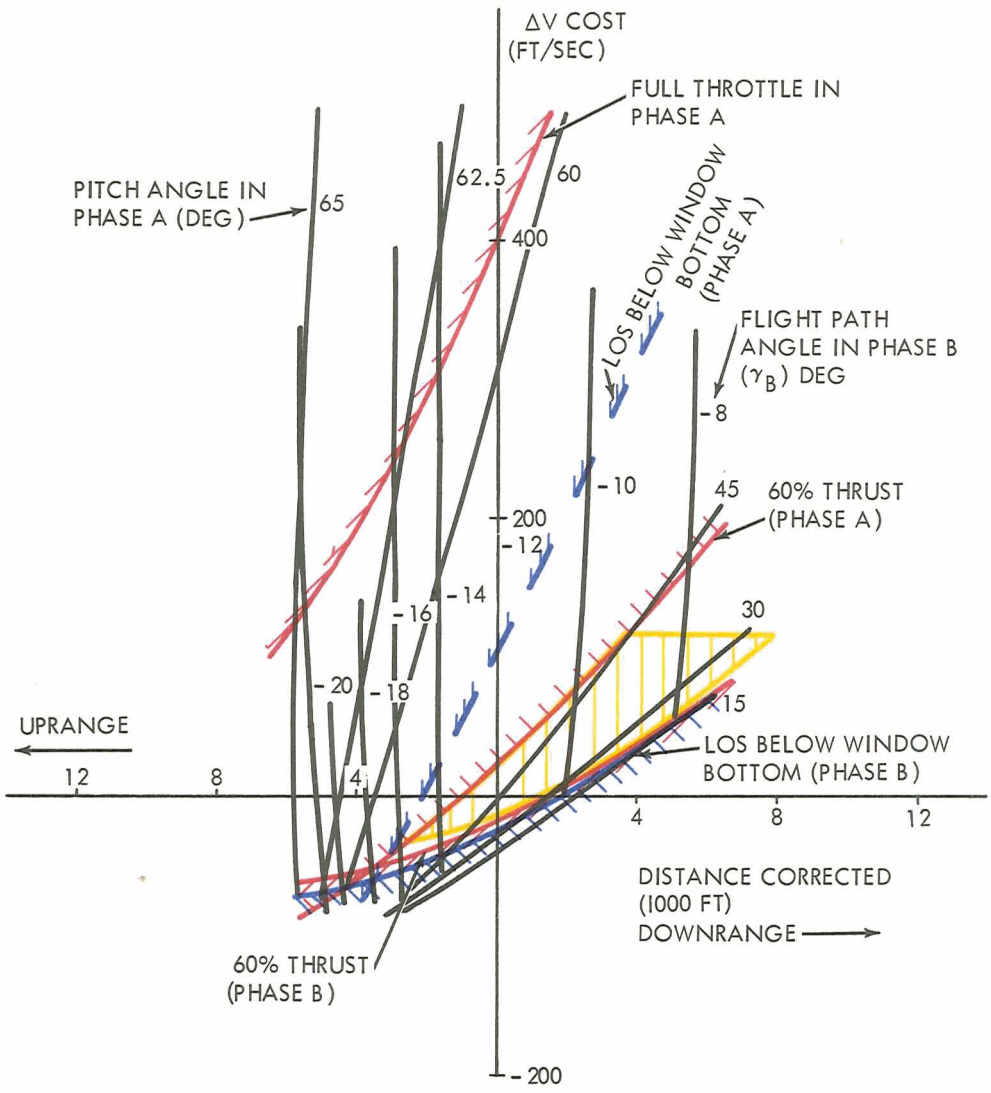

FIGURE 5 UP- AND DOWN-RANGE CORRECTIONS IN LEM DESCENT

INITIAL CONDITIONS: DISTANCE 2 1/4 N.M.
ALTITUDE 4000 FT.

FLIGHT PATH ANGLE -12 DEG CONSTANT

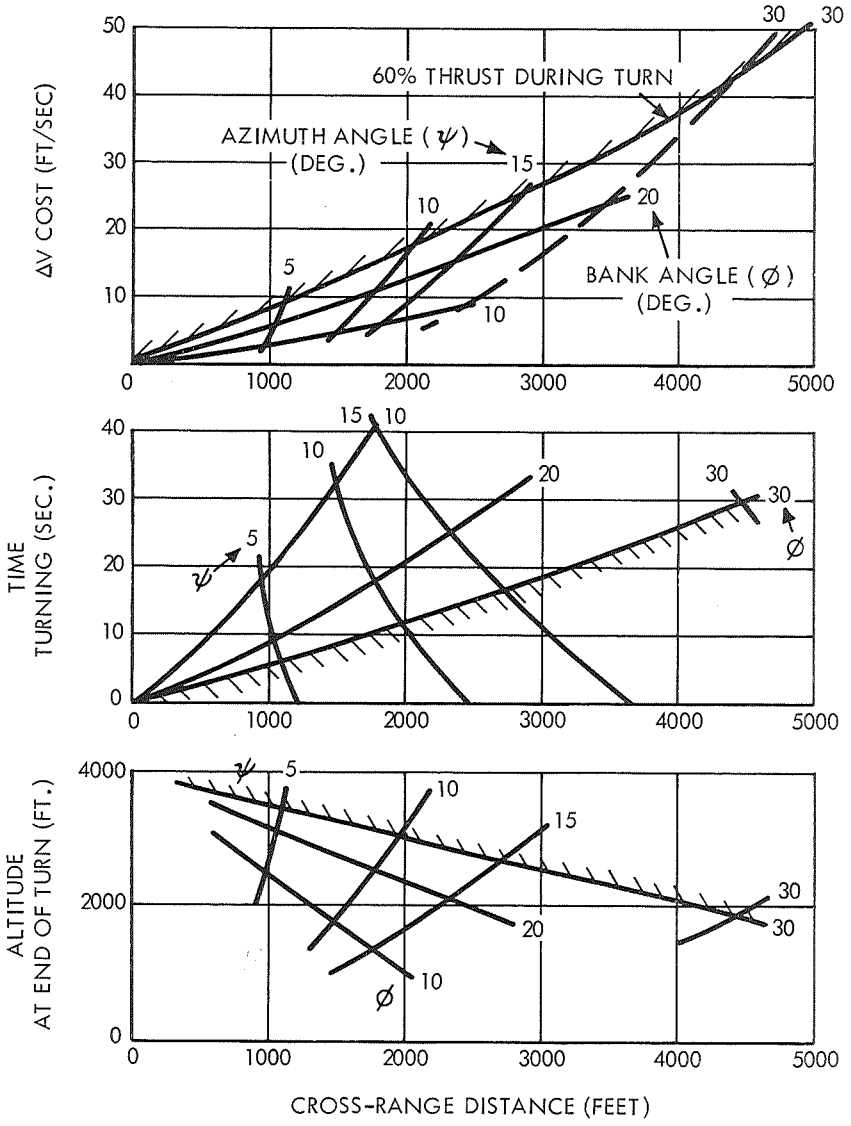

FIGURE 6 CROSS-RANGE CORRECTIONS IN LEM DESCENT